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# Chemical equilibration in Pb + Pb collisions at the SPS

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**Abstract**

An improved statistical model with excluded volume corrections and resonance decays is introduced and applied to the complete presently available set of particle ratios as measured by the various experiments at the SPS in Pb + Pb collisions. The results imply that a high degree of hadrochemical equilibration is reached at chemical freeze-out in Pb + Pb collisions.


Heavy ion collisions at ultra-relativistic energies are studied to look for signs of the production of a quark-gluon plasma phase which subsequently hadronizes. In this context one of the crucial questions is whether thermal and chemical equilibrium is achieved at some stage of the collision. Applying a statistical model which assumes equilibrium, and testing experimental data against model predictions is one way of testing reality against a thermally and chemically equilibrated fireball at the point of hadro-chemical freeze-out.

The present statistical model – like its predecessor which was presented in Refs. [1,2] – is based on the use of a grand canonical ensemble to describe the partition function and hence the density of the particles of species $i$ in an equilibrated fireball:

$$n_i = \frac{g_i}{2\pi^2}\int_0^\infty \frac{p^2\,\mathrm{d}p}{\exp\{[E_i(p)-\mu_i]/T\}\pm 1}\,, \qquad (1)$$

with particle density $n_i$, spin degeneracy $g_i$, $\hbar = c = 1$, momentum $p$, total energy $E$ and chemical potential $\mu_i = \mu_B B_i - \mu_S S_i - \mu_{I_3} I_i^3$. The quantities $B_i$, $S_i$ and $I_i^3$ are the baryon, strangeness and three-component of the isospin quantum numbers of the particle of species $i$. The temperature $T$ and the baryochemical potential $\mu_B$ are the two independent parameters of the model, while the volume of the fireball $V$, the strangeness chemical potential $\mu_S$, and the isospin chemical potential $\mu_{I_3}$ are fixed by the three conservation laws [1] for

$$\text{baryon number:}\quad V\sum_i n_i B_i = Z + N\,, \qquad (2)$$

$$\text{strangeness:}\quad V\sum_i n_i S_i = 0\,, \qquad (3)$$

and

$$\text{charge:}\quad V\sum_i n_i I_i^3 = \frac{Z-N}{2}\,. \qquad (4)$$

Here, $Z$ and $N$ are the proton and neutron numbers of the colliding nuclei. The hadronic mass spectrum

[1] These conservation laws apply strictly only for quantities which are evaluated over the complete phase space.

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used in the calculation extends over all mesons with masses below 1.5 GeV and baryons with masses below 2 GeV. This limits the temperature up to which thermal model calculations are trustworthy to $T_{max} < 185$ MeV. We note, however, that calculations with higher temperatures should anyway be considered with caution as the mass spectrum for heavier hadrons is not sufficiently well known. Maybe somewhat unexpectedly the neutron excess in Pb plays only a minor role in the determination of particle ratios.

To take into account a more realistic equation of state we incorporate the repulsive interaction at short distances between hadrons by means of an excluded volume correction. A number of different corrections have been discussed in the literature. Here we choose that proposed in Refs. [3,4]:

$$p^{\text{excl.}}(T,\mu) = p^{\text{id.gas}}(T,\hat{\mu});$$

with

$$\hat{\mu} = \mu - v_{\text{eigen}}\, p^{\text{excl.}}(T,\mu)\,. \tag{5}$$

This thermodynamically consistent approach to simulate interactions between particles by assigning an eigenvolume $v_{\text{eigen}}$ to all particles modifies the pressure $p$ within the fireball. Eq. (5) is recursive, as it uses the modified chemical potential $\hat{\mu}$ to calculate the pressure, while this pressure is also used in the modified chemical potential, and the final value is found by iteration. Particle densities are calculated by substituting $\mu$ in Eq. (1) by the modified chemical potential $\hat{\mu}$.

The eigenvolume has to be chosen appropriately to simulate the repulsive interactions between hadrons, and we have investigated the consequences for a wide range of parameters for this eigenvolume in Ref. [5]. Note that the eigenvolume is $v_{\text{eigen}} = 4 \cdot \frac{4}{3}\pi R^3$ for a hadron with radius $R$. Assigning the same eigenvolume to all particles can reduce particle densities drastically but hardly influences particle ratios. Ratios may differ strongly, however, if different values for the eigenvolume are used for different particle species.

Two different scenarios were explored: Firstly, we chose the radius of all baryons according to the charge radius of the proton, which lies at 0.8 fm, and assigned a smaller radius of 0.62 fm to all mesons, as suggested in Ref. [4]. This drastic correction reduces the thermally produced particle density in Pb + Pb collisions by a factor of seven as compared to the ideal-gas case. Furthermore, because baryons take up more space than mesons, their creation is suppressed in favor of meson production. Hence the meson to baryon ratio increases strongly. This is illustrated in Fig. 1 where we plot, for different chemical potentials, the temperature dependence of the pion/nucleon ratio and compare it to predictions from the ideal-gas scenario.

However, the nucleon-nucleon or pion-pion interaction is not repulsive at such large distances. A more physical approach is to determine, for nucleons, the eigenvolume according to the hard-core volume known from nucleon-nucleon scattering [6]. Consequently, we assigned 0.3 fm as radius for all baryons. For mesons we expect the eigenvolume not to exceed that of baryons. Therefore, to illustrate the effect, we kept the ratio of meson to baryon radii as above, implying a meson radius of 0.25 fm. As can be seen in Fig. 1, the resulting particle ratios are much closer to predictions using an ideal gas scenario; absolute yields are reduced by about 30 %. In the absence of detailed information about the meson-meson interaction at short distances we assumed for the following calculations that $R_{\text{baryon}} = R_{\text{meson}} = 0.3$ fm.

After thermal ‘‘production’’, resonances and heavier particles are allowed to decay, therefore contributing to the final particle yield of lighter mesons and baryons. Decay cascades, where particles decay in several steps, are also included. A systematic parameter regulates the amount of decay products resulting from weak decays. This allows to simulate the different reconstruction efficiencies for particles from weak decays in different experiments.

This model is now applied to Pb + Pb collisions at maximum SPS energy. We have used all data currently available. [2] We adjust the free parameters $T$ and $\mu_B$ such that they reproduce best all particle ratios available at the moment. We did not include the $2\phi/(\pi^+ + \pi^-)$ ratio in the fit, because, for this

[2] The $\bar{\Lambda}/\Lambda$ ratio from the NA49 Collaboration will be revised [7] and is therefore not included in Table 1. For the same reason we have replaced the $\Xi^-/\Lambda$ ratio from NA49 by the ratio $(\Xi^+ + \Xi^-)/(\bar{\Lambda} + \Lambda)$.

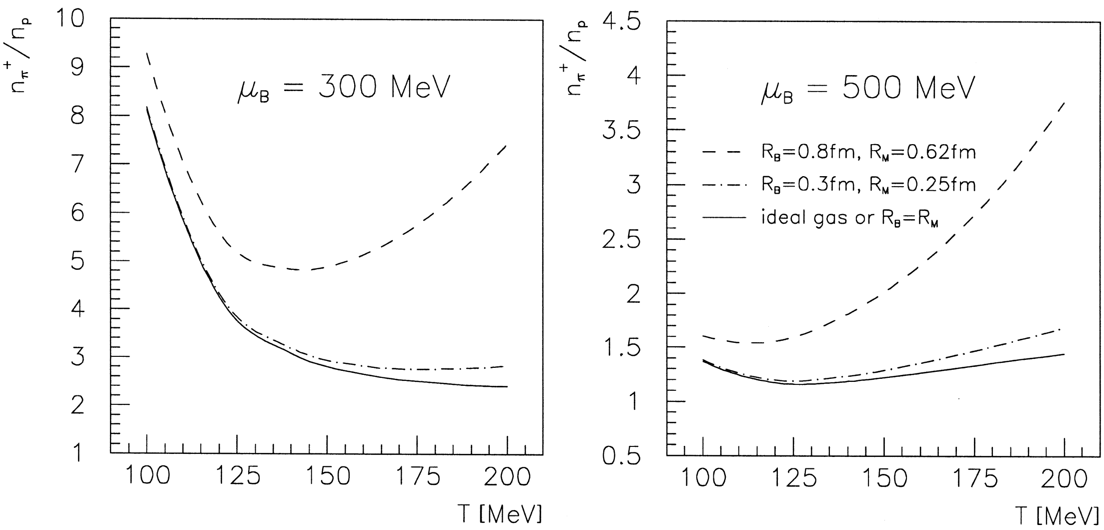


Fig. 1. The influence of different excluded-volume corrections for baryons and mesons on the $\pi^+/p$ ratio.

ratio, the currently available two experimental values exhibit a rather large discrepancy. The results for the best $\chi^2$ (see below) are shown in Table 1. For technical reasons only one of the $\bar{p}/p$ ratios (the NA49 value) was included in the fit. All experimental particle ratios are taken from data integrated over transverse momentum and integrated over rapidity $y$ to the extent data are available as shown in Table 1. Using this procedure strongly reduces the possible influence on particle ratios of dynamical effects such as hydrodynamic flow or particle production from a superposition of fireballs [8].

The criterion for the best fit was either a minimum in

$$\chi^2 = \sum_i \frac{\left(\mathscr{R}_i^{\text{exp.}} - \mathscr{R}_i^{\text{model}}\right)^2}{\sigma_i^2}, \tag{6}$$

or a minimum in the quadratic deviation

$$q^2 = \sum_i \frac{\left(\mathscr{R}_i^{\text{exp.}} - \mathscr{R}_i^{\text{model}}\right)^2}{\left(\mathscr{R}_i^{\text{model}}\right)^2}. \tag{7}$$

In the above equations $\mathscr{R}_i^{\text{model}}$ and $\mathscr{R}_i^{\text{exp.}}$ are the $i$th particle ratio as calculated from our model or measured in the experiment, and $\sigma_i$ represent the errors in the experimental data points as quoted in the experimental publications. We have used both the $\chi^2$ and the quadratic deviation measure to estimate the influence of possible systematic errors which are generally not included in the data. The deviation between these two analyses gives an indication of the accuracy of the parameters extracted from this data set.

As one can see from Fig. 2, the best fit in terms of $\chi^2$ was achieved at $T = 168 \pm 2.4$ MeV, $\mu_B = 266 \pm 5$ MeV, with $\mu_S = 71.1$ MeV and $\mu_{I_3} = -5.0$ MeV. The minimal quadratic deviation is found at $T = 164$ MeV, $\mu_B = 274$ MeV. These small differences give an indication of the systematic uncertainties of the procedure.

The overall agreement between model and data is quite good, as can be seen in Fig. 3 and Table 1. Choosing a slightly different excluded volume correction with $R_{\text{baryon}} = 0.3$ fm and $R_{\text{meson}} = 0.25$ fm yields very similar results. Using significantly larger eigenvolumes leads to much poorer agreement. Comparison between data and the model, e.g., with $R_{\text{baryon}} = 0.8$ fm and $R_{\text{meson}} = 0.62$ fm yields $\chi^2_{\text{min}} = 180$, which is roughly 5 times as large as the value shown in Fig. 2. In any case, we have excluded such large radii for independent physics reasons as discussed above.

Furthermore, using such large radii leads to an eigenvolume of all particles which would occupy

Table 1
Experimental particle ratios compared to model predictions with $R_{baryon} = R_{meson} = 0.3$ fm, $T = 168$ MeV, $\mu_B = 266$ MeV, $\mu_S = 71.1$ MeV, $\mu_{I_3} = -5.0$ MeV. $^{(a)}$: feeding from weak decays excluded, $^{(b)}$: feeding from weak decays included, $^{(c)}$: cuts exclude feeding of $\Lambda$ from $\Sigma^{\pm}$ and $\Xi$. In all three cases feeding in the model was tuned accordingly. In all other cases feeding from weak decays is assumed to be 50 %

| Particle ratio | Model | Exp. data | Exp. | y-range | Ref. |
|---|---|---|---|---|---|
| $(p-\bar{p})/h^-$ | 0.238 | $0.228(29)^{(a)}$ | NA49 | 0.2–5.8 | [9,11] |
| $\bar{p}/p$ | 0.045 | $0.055(10)^{(a)}$ | NA44 | 2.3–2.9 | [10] |
| $\bar{p}/p$ | 0.060 | $0.085(8)^{(b)}$ | NA49 | 2.5–3.3 | [11] |
| $\bar{d}/d$ | $1.78\cdot10^{-3}$ | $0.94(27)\cdot10^{-3}$ | NA44 | midrapidity | [12] |
| $\pi^-/\pi^+$ | 1.05 | 1.1(1) | NA49 | all | [13] |
| $\eta/\pi^0$ | 0.087 | 0.081(13) | WA98 | 2.3–2.9 | [14] |
| $K^0_s/\pi^-$ | 0.137 | 0.125(19) | NA49 | all | [15] |
| $K^0_s/h^-$ | 0.126 | 0.123(20) | WA97 | 2.4–3.4 | [16] |
| $\Lambda/h^-$ | 0.096 | 0.077(11) | WA97 | 2.4–3.4 | [16] |
| $\Lambda/K^0_s$ | 0.76 | 0.63(8) | WA97 | 2.4–3.4 | [16] |
| $K^+/K^-$ | 1.90 | 1.85(9) | NA44 | 2.4–3.5 | [10] |
| | 1.90 | 1.8(1) | NA49 | all | [9] |
| $\bar{\Lambda}/\Lambda$ | 0.102 | $0.131(17)^{(c)}$ | WA97 | 2.4–3.4 | [16] |
| $\Xi^-/\Lambda$ | 0.102 | $0.110(10)^{(c)}$ | WA97 | 2.4–3.4 | [16] |
| $\Xi^+/\bar{\Lambda}$ | 0.185 | 0.188(39) | NA49 | 3.1–4.1 | [17] |
| | 0.228 | $0.206(40)^{(c)}$ | WA97 | 2.4–3.4 | [16] |
| $(\Xi^+ + \Xi^-)/(\Lambda+\bar{\Lambda})$ | 0.114 | 0.13(3) | NA49 | 3.1–4.1 | [18] |
| $\Xi^+/\Xi^-$ | 0.228 | 0.232(33) | NA49 | 3.1-4.1 | [17] |
| | 0.228 | 0.247(43) | WA97 | 2.4–3.4 | [16] |
| $\Omega^+/\Omega^-$ | 0.53 | 0.383(81) | WA97 | 2.4–3.4 | [16] |
| $\Omega/\Xi$ | 0.154 | 0.219(45) | WA97 | 2.4–3.4 | [16] |
| $2\phi/(\pi^+ + \pi^-)$ | $19.0\cdot10^{-3}$ | $21(6)\cdot10^{-3}$ | NA50 | 2.9–3.9 | [19] |
| | $19.0\cdot10^{-3}$ | $12.2(13)\cdot10^{-3}$ | NA49 | all | [20] |

55% of the total volume and could therefore not be considered a ‘‘correction’’. The total fireball volume would increase to roughly 20000 fm$^3$, exceeding even the fireball volume estimated using pion interferometry. As discussed in Refs. [21,22], the total fireball volume in central Pb + Pb collisions at thermal freeze-out should be about 13500 fm$^3$.

In the small excluded volume scenario with $R_{baryon} = R_{meson} = 0.3$ fm the fireball volume of 2800 fm$^3$ is considerably smaller. The corresponding pion density is then 0.60 pions/fm$^3$, significantly exceeding the measured pion density of roughly 0.12 pions/fm$^3$ [22]. This is not surprising, however, as the calculated pion density of 0.6/fm$^3$ is determined at chemical freeze-out corresponding to $T = 168$ MeV. If one lets this fireball expand isentropically to 125 MeV, the temperature roughly corresponding to thermal freeze-out as indicated by particle spectra [23] and two-pion correlations [21] the corresponding pion density is 0.084/fm$^3$, close to the experimental value. Note that the calculated pion density would further increase by about 30% if one were to reduce to zero the meson radius in the excluded volume correction.

The present results, in particular those involving multi-strange baryons, imply that no separate strangeness suppression factor is needed to describe the available Pb + Pb data at SPS energy. In fact, the mean value of the experimental to calculated yields ratios involving $\Delta S =$ 1, i.e. those which are sensitive to a possible overall strangeness suppression, is $0.96 \pm 0.05$, consistent with unity. This conclusion differs from that reached in a recent investigation [24] where, however, only a very restricted set of ratios was used for comparison with thermal model predictions. An interesting anomaly would arise if the $\phi$-meson yield converges to the low value reported by the NA49 Collaboration (see Table 1), since this meson carries two units of hidden

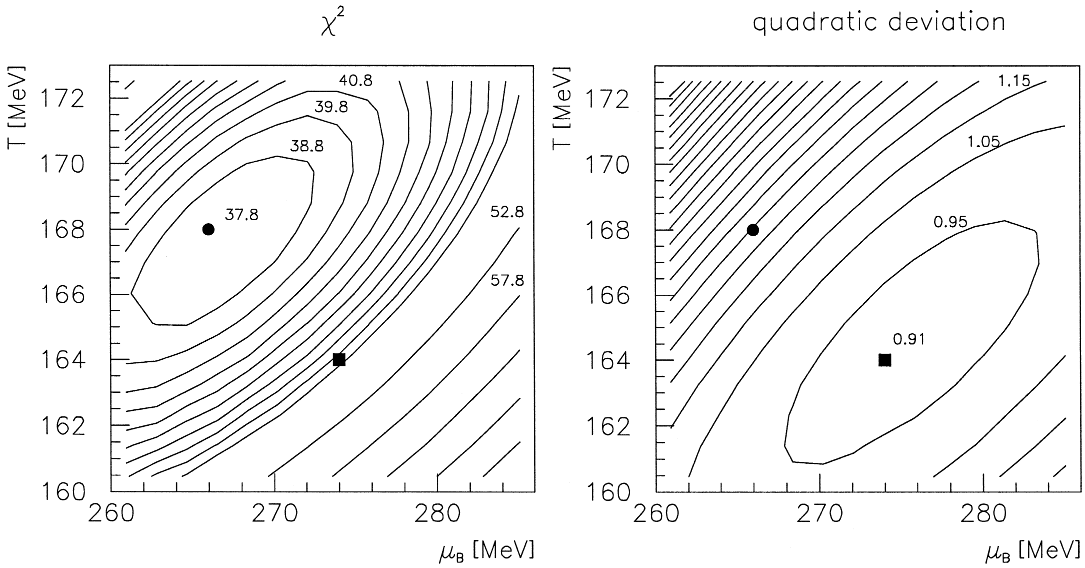


Fig. 2. $\chi^2$ and quadratic deviation for the comparison between model particle ratios and data explored over a wide range of parameters. The dot represents the parameter set with minimum $\chi^2$, the square the set with minimum quadratic deviation.

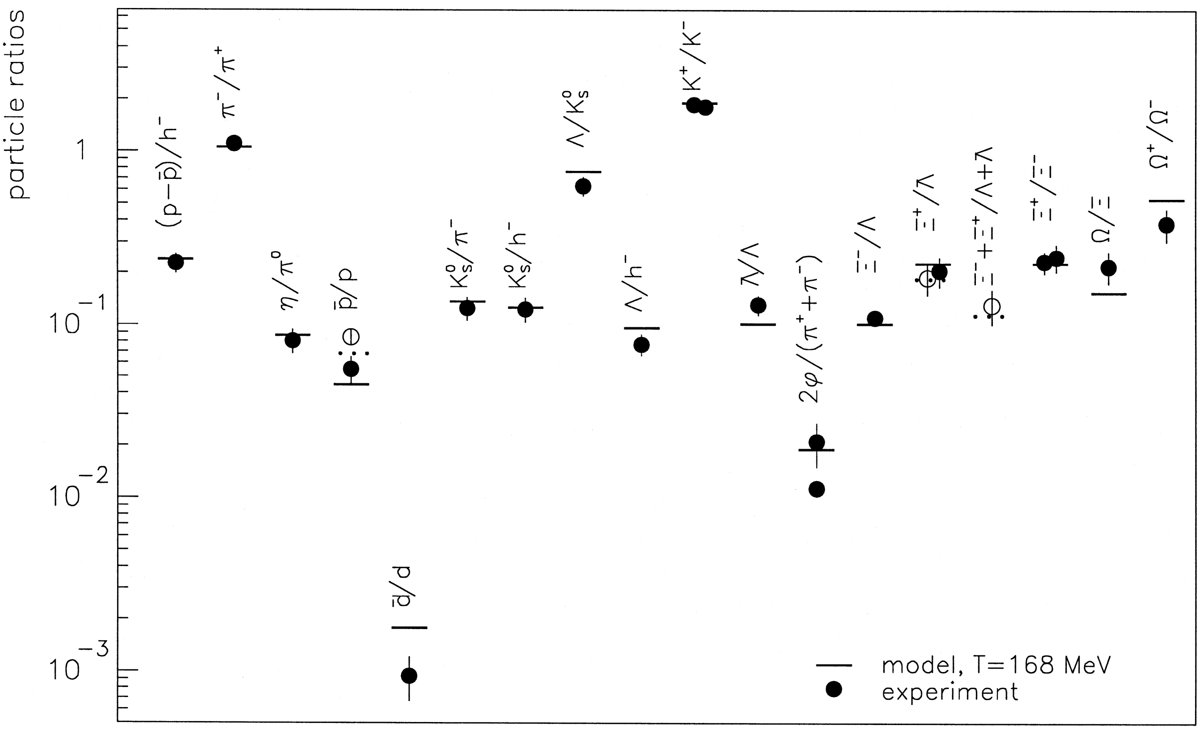


Fig. 3. Comparison between model and experimental particle ratios. For experimental data, errors and information about feeding corrections, see Table 1 and references therein.

strangeness. To reconcile this with the results of the WA97 Collaboration on cascade or omega-baryon production would be a challenge.

We further note that the improved model discussed here was also applied to the AGS data collected in Ref. [1]. The best fit, obtained for $R_{\text{baryon}} = R_{\text{meson}} = 0.3$ fm, yields $T = 125(^{+3}_{-6})$ MeV and $\mu_B = 540 \pm 7$ MeV, well in line with the calculations reported in Ref. [1]. Here, the corresponding $\pi^+$ and proton densities of $0.051/\text{fm}^3$ and $0.053/\text{fm}^3$ agree well with those estimated from particle interferometry [25,26] ($0.058/\text{fm}^3$ and $0.063/\text{fm}^3$, respectively) implying that, at AGS energy, thermal and chemical freeze-out take place at nearly identical temperatures.

The good agreement between the predictions of the thermal model and the measured particle ratios implies that thermal and chemical equilibrium is established (or at least closely approached) in the fireball at hadrochemical freeze-out. Furthermore, it is interesting to note that the resulting temperature and chemical potential values are very close to where we believe is the phase boundary between hadronic matter and the quark-gluon plasma [27]. It is, therefore, quite probable that the system crosses this phase boundary shortly before it freezes out hadrochemically.

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